

**LEBANESE AMERICAN UNIVERSITY**  
Department of Computer Science and Mathematics  
**Discrete mathematical Structures I**  
**Final Exam Fall 12/13 (January 26, 2013)**

Name: Solutions

ID:

<u>Question Number</u>	<u>Grade</u>
1. 6%	
2. 4%	
3. 5%	
4. 4%	
5. 6%	
6. 6%	
7. 5%	
8. 8%	
9. 7%	
10. 3%	
11. 4%	
12. 6%	
13. 6%	
14. 5% 17	
15. 8%	
16. 6%	
17. 6%	
18. 5%	
Total	

1. (6%) Consider the Predicate  $P(m, n) : m \mid n$ , where the domain is in  $\mathbb{N}^*$ . Translate into a MEANING-FUL English statement the following:

FUL English statement the following

(e)  $\exists m : (\forall n : P(m, n))$

there is an integer half

divides all integers.

(b)  $(P(m, n) \wedge m \geq n) \rightarrow m = n$

If  $m \neq n$  and  $m \geq n$ , then  
 $m$  must equal  $n$

(c)  $\exists m : \exists n \sim P(m, n)$

There is a certain integer  $m$  that does not divide a certain given integer  $n$ .

2. (4%) Consider the statement: "Some LAU students do not drink coffee". Translate into symbols using two different Predicates  $P$  and  $Q$ : Use the domain: all **people** at LAU.

$x \in$  people in L A U.

@CAC

$\exists x: [P(x) \wedge x \text{ drinks coffee} \rightarrow Q(x)]$

3. (5%) Show whether or not the two propositions  $p \rightarrow (q \rightarrow r)$  and  $(p \rightarrow q) \rightarrow r$  are equivalent. (DO NOT USE TRUTH tables.)

1

$\vdash \neg b \rightarrow (a \leftrightarrow b)$

$\neg(\rho \rightarrow q) \vdash r$ :  $F$ .  $\neg(\rho \rightarrow q) \vdash T$ ,  $r : F$

4. (4%) Show that  $A - (B \cap C) = (A - B) \cup (A - C)$

$$\begin{array}{r} \textcircled{B} \\ \textcircled{A} \end{array}$$

$$A - B = 1, 4 \quad (A - B) \cup (A - C) = 1, 2, 4 \\ A - C = 1, 2$$

1 Same  $\Rightarrow$  = ✓

5. (6%) Let  $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}$  be defined by  $f(m, n) = (m+1, n^2)$

(a) Is  $f$  1-1?

$$\begin{aligned}f: \dim & f(2, 3) = f(2, -3) \\&= (3, 9) = (3, 9) \\&\text{although } (2, 3) \neq (2, -3) \Rightarrow \cancel{\text{Not 1-1}}\end{aligned}$$

(b) Is  $f$  onto?

$$No, \text{ since } \#(m, n) \text{ s.t. } f(m, n) = (0, -5)$$

6. (6%) Under what conditions do we have that  $[2x] = 2[x]$  for  $x \in \mathbb{R}$  explain.

$$\begin{aligned}\text{Case 1: } x &= n+d & d &\in [0, 1/2) \\ \Rightarrow x &= 2n + 2d & 2d &< 1 \\ \Rightarrow \lfloor 2x \rfloor &= 2n = 2 \lfloor x \rfloor.\end{aligned}$$

$$\text{Case 2: } x = n+d; \quad \frac{1}{2} \leq d < 1 \\ \lfloor 2x \rfloor = \lfloor 2n + 2d \rfloor = 2n + 1$$

$$\text{Answer: } \lfloor 2x \rfloor = 2 \lfloor x \rfloor \text{ when } x = n+d \quad 0 \leq d < 1/2$$

7. (5%) Fill in the blanks  $(AB)^T = ..$  where  $A$  is an  $m \times n$  matrix and  $B$  is an  $n \times p$  matrix.

$$B^T A^T$$

8. (8%) Show using Mathematical Induction that for all integers  $n$ , we have:  $1 * 2 + 2 * 3 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

(i) Basic Step:

$$n=1 \quad 1 * 2 = \frac{1(2)(3)}{3} \checkmark$$

2) Ind. Step: Assume  $P(k) : T$ , show

$Q(k+1) : T$ .

$$\text{Assume: } 1 * 2 + 2 * 3 + \dots + k(k+1) = \frac{k(k+1)(k+2)}{3}$$

Show:

$$1 * 2 + 2 * 3 + \dots + k(k+1) + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$$

$$\frac{k(k+1)(k+2)}{3} + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$$

$$k(k+1)(k+2) + 3(k+1)(k+2) = ? (k+1)(k+2)(k+3)$$

✓ Verified

9. (7%) Consider  $m = 41, n = 18$

(a) Use the Euclidean Algorithm to find  $d = \gcd(m, n)$

$$41 = 2(18) + 5$$

$$18 = 3(5) + 3$$

$$5 = 3(1) + 2$$

$$3 = 2(1) + 1 \leftarrow \text{gcd} = 1$$

$$2 = 2(1) + 1$$

(b) Deduce  $d$  as a linear combination of those two numbers: in other words write  $d$  as:  $d = sm + tn$ , for some  $s, t \in \mathbb{Z}$

$$\begin{aligned} 1 &= 3 - 2 = 3 - (5 - 3) = 2(3) - 5 \\ &= 2[18 - 5(3)] - 5 \\ &= 2(18) - 7(5) \\ &= \cancel{2(18)} = 2(18) - \cancel{7}(41 - 2(18)) \\ &= \cancel{2(18)} - \cancel{7}(41) \end{aligned}$$

10. (3%) Consider the statement: "A function is a relation" ... and the statement: "a relation is a function"?

Which of them is correct. In case one is incorrect, provide a counterexample.

A function is a relation.

But a relation is not a function since one input can have several outputs

1 R<sub>2</sub>, 1 R<sub>3</sub>, 1 R<sub>4</sub> ...

11. (4%) In how many ways can you place 67 non-distinguishable toys in 8 baskets?

$$\frac{(67+7)!}{67! \cdot 7!}$$

12. (6%) If  $a \equiv b \pmod{n}$  and  $c \equiv d \pmod{n}$ , show that  $ac \equiv bd \pmod{n}$

$$a = nq + b$$
$$c = nq' + d$$

$$\begin{aligned} ac &= \cancel{nq} (nq + b) (nq' + d) \\ &= n^2qq' + nqd + nq'b + bd \\ &= n ( \dots ) + bd. \end{aligned}$$

$$ac \equiv bd \pmod{n}.$$

13. (6%) Fill in the blanks below: Among 400 people,

- (a) ... have the same birthday

at least 2

- (b) ... were borne on the same day of the week

at least 52 : since  $7 * 57 - 1 = 399$

- (c) ... were borne in the same month

$33 * 12 = 396$

at least 34 since

14. (5%) Find, if possible, a relation on the set  $A = \{1, 2, 3\}$  that is (AR), (S) and (T).

The only possibility is :  $R = \emptyset$

since if  $1 \sim 2 \Rightarrow$



ONLY  $R = \emptyset$

$\Rightarrow$  not AR.

15. (8%) Consider the relation  $R$  on  $\mathbb{Z}$ .  $mRn$  if and only if  $m^2 - n^2$  is divisible by 6.

(a) Show, using the most efficient method that  $R$  is reflexive, symmetric and transitive.

Pick  $f : f(m) = m^2 \text{ Mod } 6$  then  
 $mRn \Leftrightarrow f(m) \sim f(n) \Rightarrow R: \text{eq. rel.} \Rightarrow$   
 $\checkmark$  (a) (S) (T)

- (b) Find all the equivalence classes determined by  $R$ .

$m \text{ Mod } 6$	$m^2 \text{ Mod } 6$
0	0
1	1
2	4
3	3
4	4
5	1

4 eq. classes total

$$\begin{cases} 6k = [0] \\ 6k+1; 6k+5 = [1] \\ 6k+2, 6k+4 = [2] \\ 6k+3 = [3] \end{cases}$$

16. (6%) Consider the relation  $R$  on  $A = \mathbb{Z}$  where  $aRb$  if and only if  $a \leq 3b$ . What properties does the relation  $R$  have?

1)  $(R)$  since  $a \leq 3(a)$

2)  $\not(R)$  since  $1R10 (1 \leq 3(10))$  but  $10 \not R 1$   
 $\quad \quad \quad$  since  $10 \neq 3$

3)  $\overline{R}(\tau) \text{ since } 5R2 \text{ and } 2R1 \text{ but } 5 \not R 1$

but  $5 \not R 1 \quad 5 \neq 3$

17. (6%) Consider the relation  $R$  on  $A = \{1, 2, 3, \dots, 1000\}$  where  $mRn$  iff  $m \leq n$

(a) How many non-zero entries does the matrix  $M_R$  representing the relation have?

Diagonal and under main if!

$$\rightarrow 499, 500 + \underbrace{100}_{\text{diagonal}} = \boxed{500, 500}$$

(b) Same question for the relation  $S$  on  $A$  given by  $mSn$  iff  $m \neq 0$

$$(1000)^2 - 10^6$$

Since all are nonzero ||.

18. (5%) Let  $A = \mathbb{N}^*$  and  $R$  be the relation on  $A$  given by  $mRn$  iff  $m|n$ . Show that  $(A, R)$  is a POSET.

y  $R$  is  $(R)$  since  $m|n$

2)  $R$  is ~~(Ref)~~ not (S)  $3 | 6$   $6 \nmid 3$

3)  $R$  is (T)  $a | b$   $b | c \Rightarrow a | c$ .

$\therefore R$ : partial order.